**Thinking Like a Physicist**

**Planck Length Thought Experiments**

Everything we know about the universe at a fundamental level is an idea from either *quantum theory* (more precisely, quantum field theory) or *gravity theory* (more precisely, Einstein’s special and general theories of relativity). One of the greatest unsolved mysteries in theoretical physics today is how these two theories can work together in a single, unified theory of *quantum gravity*.

Just as *quantum* theory is needed to understand how nature works at the microscopic scale of atoms (about m), it is widely believed that a theory of *quantum gravity* will be needed to understand how nature works at the *ultra*microscopic scale called the *Planck length* (about m). At this tiny scale it is believed that nature behaves in ways that are still deeply mysterious, that will push us to entirely new ways of thinking about the ultimate nature of reality.

Working in small groups, you will conduct **two** thought experiments that lead to this conclusion.

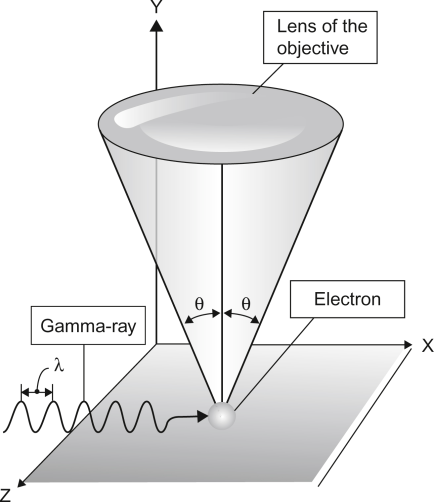
**Thought Experiment #1―Dimensional Analysis**

While we do not yet have a generally agreed upon theory of quantum gravity, any prediction such a theory would make must involve quantum theory, special relativity, and general relativity, and hence must involve the following three fundamental constants of nature:

1. **Planck’s constant, kg m2/s**. Planck’s constant is the fundamental constant associated with quantum theory―it “sets the scale” for quantum effects. For example, a light wave of frequency is actually a shower of quantum particles called photons, each with energy . The scale of this quantum “graininess” is set by the numerical value of . As is customary in more advanced physics, we will use what’s called the *reduced Planck’s constant*:  **kg m2/s** (pronounced “h-bar”), instead of .
2. **Speed of Light, m/s.** The speed of light is the fundamental constant associated with special relativity―it “sets the scale” for all effects related to velocity (motion through spacetime). Every fundamental effect that depends on an object’s speed, , actually depends not on , but on the *ratio* of to . This ratio is a *dimensionless* number measuring the object’s speed as a fraction of the universal speed limit.
3. **Newton’s constant, m3/kg s2.** Newton’s constant is the fundamental constant associated with gravity theory―it “sets the scale” for gravitational effects. For example, when we drop an object it appears to accelerate toward the ground at a certain rate. That rate is set by the numerical value of . If was larger in our universe, the object would accelerate at a greater rate.

The most basic prediction we can make is the *length scale* at which quantum gravity effects should become important. **Can you put the three constants , , and together in a combination that has the *dimensions of length* (i.e., units of metres), and hence guess a formula for this length scale?** Substitute numbers into your formula; do you get the *Planck length* of about m quoted above?

**Thought Experiment #2―Generalized Uncertainty Principle**

The Heisenberg Uncertainty Principle (HUP) reads

Objective Lens

To understand what it means, consider trying to locate the position of an electron as accurately as possible by shining light on it, and viewing the scattered light through a microscope (see diagram). It is a simple fact of the *wave nature* of light that the image of the electron in the microscope will be *fuzzy*―the electron’s position can be determined only to an accuracy about equal to the wavelength, , of the light being used. Shorter wavelength results in greater accuracy. More precisely, the uncertainty in the electron’s position is given by

Light

Electron

where is the angle of the cone of light captured by the objective lens of the microscope. is called the *resolving power* of the microscope. (If you don’t already know this formula, just take it for granted.)

However, according to quantum theory, light also has a *particle nature*. It behaves as if it is composed of a shower of particles called *photons*, each with energy and momentum . Like a collision between two billiard balls, a photon that hits the electron and scatters into the objective lens will impart some momentum to the electron. What’s of interest is not the *amount* of momentum, but rather the *range* of momentum, i.e., the *uncertainty* in the amount of momentum imparted to the electron. If the photon scatters toward the *right* edge of the objective lens, it will impart *less* momentum to the electron than if it scatters toward the *left* edge of the objective lens. Convince yourself that the *range* of this difference in momentum is approximately

If the objective lens is very narrow ( is very small), any photon we see in the microscope must have scattered at a very precise angle (almost exactly degrees up in the diagram), imparting a *definite* amount of momentum to the electron. *Increasing* has the advantage of *decreasing* the uncertainty in the electron’s position ( decreases, i.e., the resolving power of the microscope improves), at the expense of *increasing* the uncertainty in the electron’s momentum ( increases).

Similarly, if we use light of shorter wavelength to improve the resolving power of the microscope ( *decreases*), the photon momentum increases, which increases the *uncertainty* in the electron’s momentum ( *increases*). Decreasing increases and vice versa.

However, the *product* of the two uncertainties is a universal constant:

This is the Heisenberg Uncertainty Principle.

According to the HUP, there is no limit to the precision with which we can measure the electron’s position (i.e., we can make as small as we want), *provided* we allow a large uncertainty in the electron’s momentum (a large ). We can do this by using shorter and shorter wavelength light. But this does not take into account the *gravitational* effects of a photon. We will do so in the following questions, and discover that we *cannot* make as small as we want. In fact, we cannot make it smaller than about the Planck length.

**Questions:**

1. According to quantum theory, a photon of frequency has energy . Use the universal wave relation for light, , to express the photon energy in terms of its wavelength.
2. According to special relativity, the energy and mass of any entity are related by . Use this, and your result from Question #1, to compute the effective mass, , of a photon. (A photon has zero rest mass, but it has energy, and this energy has an effective mass that exerts a gravitational force in exactly the same way a massive object does.)
3. According to general relativity, will exert a gravitational force on the electron in our Heisenberg microscope thought experiment. This force will *accelerate* the electron in some random direction, depending on how the photon approaches the electron. Use Newton’s law of gravity to estimate the *magnitude* of this acceleration, . (Assume that the photon and the electron are at some average effective distance, , apart during the scattering process.)
4. If an object starting at rest experiences an acceleration for a time , it will be displaced by a distance . Ignoring the factor of , use this formula to estimate the magnitude of the random gravitational displacement, , caused by this acceleration. (Assume that the scattering process takes some average effective time, , to occur.)
5. Your formula for should involve the ratio of to . This ratio is a distance divided by a time, and hence a speed. The only relevant speed in the scattering process is the speed of the photon, i.e., the speed of light. Simplify your expression for by replacing with .
6. Ignoring the numerical factor of , simplify your expression for further by expressing it in terms of the Planck length, (from Thought Experiment #1), and the photon wavelength, . Finally, replace with the approximate Heisenberg uncertainty in position, .
7. Add this *gravitational* contribution to the electron’s uncertainty in position to the Heisenberg uncertainty in position to arrive at a *Generalized Uncertainty Principle* (GUP):
8. What’s interesting about the GUP is that *cannot* be made as small as we want by allowing to become arbitrarily large (why not?). Sketch a graph of and (as functions of ), as well as the *sum* of these two functions. Notice that this curve has a *minimum* value, , which occurs for , i.e., .

**So What?**

Combining ideas from quantum theory, special relativity, *and* general relativity (gravity), we learn that we cannot resolve the position of a point particle to better than about the Planck length. This form of the GUP has been obtained in numerous ways, ranging in sophistication from our simple thought experiment above to several versions of superstring theory; it appears to be a rather general result of combining quantum theory with gravity theory.

According to the GUP, we cannot measure a particle’s position more accurately than the Planck length. The Planck length seems to represent some kind of *minimum physically meaningful distance*. If such a distance exists, it suggests that, in a theory of quantum gravity, space might not be smooth “all the way down”. Perhaps space (and possibly time) are somehow “grainy” at the ultramicroscopic Planck scale. This has provided a powerful clue to researchers thinking about possible theories of quantum gravity.

**Discuss some implications of this thought experiment.**